Existence and properties of minimal solutions of elliptic problems with sign-changing nonlinearities

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In this talk we show the existence of minimal positive solutions of the problem

$$\begin{cases} \triangle u(x) + f(x, u(x)) + g(x, u(x))x \cdot \nabla u(x) = 0 \quad \text{for} \quad x \in \Omega_R \\ \lim_{\|x\| \to +\infty} u(x) = 0 \end{cases} \tag{0.1}$$

in the set $\Omega_R = \{x \in \mathbb{R}^n, \|x\| > R\}$ for $R > 1$ and $n \geq 3$ with positive g satisfying an additional condition and f with various growths with respect to the second variable.

We divide our considerations in two cases: when f is nonnegative at the origin (so-called "positone problem") and when f may change its sign. The latter case allows us to discuss also so-called "semipositone problem" (with f negative at the origin).

By using the subsolution-supersolution method, we find a minimal positive solution u of (0.1), which is between a nonnegative subsolution w and a positive supersolution v and $u = v$ on $\partial\Omega_R$. We construct a supersolution v via fixed-point method (Krasnosel'skiĭ-Guo fixed point theorem in cones).

We formulate conditions guaranteeing the existence of solutions u_1 and u_2 of (0.1) such that $0 < \|u_1\| \leq d \leq \|u_2\|$, where d is a given positive number.

References

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