Anisotropic Operators in Generalized Morrey Spaces

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In this work, we investigates the boundedness and continuity properties of anisotropic sublinear operators of Calderón-Zygmund type, anisotropic Riesz potentials, and fractional maximal operators within the framework of vanishing generalized Morrey spaces [4]. These spaces provide a flexible setting to capture fine local behavior of functions in non-homogeneous and directionally scaled contexts, relevant to anisotropic harmonic analysis [1].

We introduce a broad class of growth functions that satisfy appropriate structural and integrability conditions. Under these assumptions, we establish that the considered operators are bounded from the vanishing generalized Morrey space $V\mathcal{M}^{p,\phi}(\mathbb{R}^n)$ into itself or into related function spaces [4], [5]. Furthermore, we prove that the space $V\mathcal{M}^{p,\phi}(\mathbb{R}^n)$ is complete and that smooth compactly supported functions form a dense subset [7], [8], ensuring the robustness of the functional analytic framework.

Our approach is based on deriving precise pointwise modular estimates that control the operator behavior at small scales. These estimates allow for refined control over the action of singular and potential-type operators in anisotropic settings [2], [3]. The proofs rely on harmonic analysis techniques adapted to the vanishing Morrey space structure and anisotropic geometry [4], [6].

References

[1] M. Bramanti, M.C. Cerutti, Commutators of Singular Operators on Homogeneous Spaces, *Boll. Un. Mat. Ital. B*, **II**2003, 843-883.
[2] E.B. Fabes, N.M. Rivière, Commutators of singular integrals, *Studia Math*, **10-7**1996, 9-17.
[3] D. Gilbarg, N.S. Trudinger, *Elliptic Partial Differential Equations of Second Order*, Springer-Verlag, 1983.
[4] N. Samko, potential and singular operators in vanishing generalized Morrey spaces, *J. Global Optim*, **57**2013, 1385–1399.
[5] L. Softova, Singular integrals and commutators in generalized Morrey spaces, *Acta Math. Sin. (Engl. Ser.)*, **22-3**2006, 757–766.
[6] E. Nakai, Hardy-Littlewood maximal operator, singular integral operators and the Riesz potentials on generalized Morrey spaces, *Math. Nachr*, **166**1994,95–103.
[7] C.T. Zorko, Morrey space, *Proc. Of The Amer. Math. Soci.*, **98**1986, 586–592.
[8] L. Caso, R. D'Ambrosio, L. Softova, Generalized Morrey Spaces over Unbounded Domains, *Azerbaijan Journal of Mathematics*, **10**2020.