

Positive solutions to singular elliptic problems in exterior domains - existence and properties

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We present the results regarding the existence of positive solutions of the following singular problem

$$\begin{cases} \Delta u(x) + f(x, u(x)) - b(x)(u(x))^{-1} \|\nabla u(x)\|^2 + k(x, x \cdot \nabla u(x)) = 0, & x \in \Omega_R, \\ \lim_{\|x\| \rightarrow \infty} u(x) = 0, \end{cases} \quad (1)$$

where $\Omega_R = \{x \in \mathbb{R}^n, \|x\| > R > 2\}$, $n \geq 3$, functions f , b , k are sufficiently smooth and $b(x) > 0$.

In the first part of the presentation, we discuss history of similar problems ([1], [2]) and we establish classical solutions of a problem without a singular part. Our main tool is the Noussair-Swanson theorem concerning the sub-supersolution approach ([4]).

In the second part, we show the existence of a positive solution to (1) using sub-supersolution method based on the work of S.Cui ([3]) for bounded sub-domains as well as the unbounded domain approximation method combined with some classical convergence procedure ([4], [5]). Solution is equal to both supersolution and subsolution for $\|x\| = R$.

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