Existence results for semi-linear differential equations with nonlocal boundary value conditions of Dirichlet type

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Let us consider the following boundary value problem

$$\begin{cases} -\Delta u = f(\boldsymbol{x}, u) & \text{in} \quad \Omega, \\ u = \displaystyle\int_{\Omega} K\left(\cdot, \boldsymbol{\xi}\right) h\left(\xi, u(\boldsymbol{\xi})\right) \, \mathrm{d}\boldsymbol{\xi} & \text{on} \quad \partial\Omega, \end{cases} \tag{0.1}$$

where f, h, K are suitable functions and $\Omega \subset \mathbb{R}^d$ is open and bounded with a Lipschitz boundary. We have studied examined the solvability of the problem (0.1). Results concerning the existence of solutions to this type of problems can be found, among others, in [2, 3], the analysis is performed in the space of continuous functions, primarily using Krasnoselskii fixed-point theorem in a cone. These results have been accepted for publication (see [1]) .

References

[1] M. Bełdziński, M. Galewski, I. Kossowski, Existence results for semi-linear differential equations with nonlocal boundary value conditions, accepted for publication in *Topological Methods in Nonlinear Analysis*.

[2] S. Biagi, A. Calamai, G. Infante, Nonzero positive solutions of elliptic systems with gradient dependence and functional BCs, *Advanced Nonlinear Studies*, 20(4) (2020), 911–931.

[3] I. Kossowski, Radial solutions for nonlinear elliptic equation with nonlinear nonlocal boundary conditions, *Opuscula Mathematica*, 43(5) (2023), 675–687.