# On the regularity of stationary harmonic functions whose Laplacian is a Radon measure

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Let $\Omega \subset \mathbb{R}^n$ be a domain. We say that the function $h \in H^1(\Omega)$ is stationary harmonic if

$$\operatorname{div} T_h = 0 \tag{$*$}$$

in the sense of distributions, where $T_h$ is the stress-energy tensor associated to the Dirichlet energy, defined by

$$T_h = \nabla h \otimes \nabla h - \frac{1}{2}|\nabla h|^2 Id.$$

The equation $(*)$ is understood as $n$ equations of the form $\operatorname{div}(T_h)_i = 0$ $(i = 1, \ldots, n)$, one for each row of the matrix $T_h$. We will show that if the function $h$ satisfies $(*)$ and its distributional Laplacian $\Delta h$ is a Radon measure, then $h$ cannot be of class $C^1$ unless $h$ is a harmonic function. It follows that $\mu = \Delta h$ cannot be of the form $\mathcal{H}^\alpha \llcorner_\Sigma$, where $\alpha > n-1$ and $\Sigma$ is a Borel set of nonzero, finite $\mathcal{H}^\alpha$ measure.

The structure of possible measures $\mu = \Delta h$ such that $h$ satisfies the equation $(*)$ was previously considered in two dimensions in the paper [1]. This work (in progress) is a continuation that tries to extend those results to more dimensions.

## References

[1] Remy Rodiac, *Regularity Properties of Stationary Harmonic Functions Whose Laplacian is a Radon Measure*, SIAM J. Math. Anal., Vol. 48, Iss. 1 (2016).